B 68 11046

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

SUBJECT: Reliability Testing in the Presence of a Decreasing Failure Rate - Case 101

DATE: November 15, 1968

FROM: B. J. McCabe

ABSTRACT

This memorandum summarizes some statistical techniques to be used in reliability tests of equipment whose performance is known to improve as the time during which it has been in operation increases. A more detailed account is given in TM-68-1033-8.

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MEMORANDUM FOR FILE

This memorandum summarizes some statistical techniques to be used in reliability tests of equipment whose performance is known to improve as the time during which it has been in operation increases. For a more detailed account see the reference [1].

Let S_K denote the time at which the K-th failure of the system occurs, where time is measured from the system's installation and repair times are ignored. Then $S_K - S_{K-1} = X_K$ is the time between the failures numbered K-1 and K. These random variables X_K are assumed to be independent.

Let N(t) be the number of failures occurring before time t. Thus N(t) = K if and only if $S_K < t \leq S_{K+1}$. Let H(t) = E(N(t)), the mathematical expectation of N(t) and let h(t) = H'(t). The function h is called the failure rate.

We suppose that the times between failures X_K are tending to be longer, as the system wears in, and eventually stabilize so that at some unspecified time it may be assumed that there is such a thing as a fixed mean time between failures μ, $0 < \mu < \infty$. Therefore, it is assumed that h(t) is a non-increasing function such that $\lim_{t\to\infty} h(t) = 1/\mu$.

Some estimates for h, μ, and the point $t^*(\varepsilon)$ at which h first comes within ε of its eventual limit $1/\mu$ will now be listed.

1. First estimate for h.

Let K be a function satisfying:

$$K \geq 0,\ K(-x) = K(x),\ \int_{-\infty}^{\infty} K(x)dx = 1 \quad , \quad \text{and}$$

$K(x) = 0$ for all x outside some finite interval. Then we set

$$\hat{h}_K(t) = \int_0^\infty K(x-t)\,dN(x)$$

$$= \sum_{k=1}^{\infty} K(S_k - t) \quad .$$

$\hat{h}_K$ may be taken as an estimate for h, but a smoothing operation which exploits the fact that h is assumed to be decreasing will improve the estimate.

Let $\bar{h}$ coincide with $\hat{h}$ until the first point, say t_o, at which h increases. Then redefine $\bar{h}$ on the intervals to the left and right of t_o to be the average of the two values assumed by $\hat{h}$ on these intervals. If this succeeds in making $\bar{h}$ decreasing then proceed to the next point of increase for $\hat{h}$. Otherwise further averaging may be carried out on the intervals to the left of t_o, as many as are necessary to make $\hat{h}$ decreasing. See [1, p. 11] where an example is worked out.

2. Second estimate for h.

Let $h^*(t) = 1/X_{n+1}$ for $S_n < t \leq S_{n+1}$. A convenient smoothing for h^* is as follows: if h^* increases at t_o, say from y_1 to y_2, then let h^* equal $2/(y_1^{-1} + y_2^{-1})$ on the intervals to the left and right of t_o. Perform this operation as many times as are necessary to make h^* decreasing, and let this new estimate be denoted by h^* also.

It is hoped that a future paper will be devoted to statistical properties of h^*. It can be stated now, however, that with some additional assumptions,

$$E(1/h^*(T)) \to \mu \quad \text{as } T \to \infty \; .$$

Therefore $1/h^*(T)$ is an asymptotically unbiased estimate for μ.

3. Estimating the beginning of the stable period.

Let $\varepsilon > 0$ be given, and suppose that at time T one wishes to estimate $t^* = t^*(\varepsilon)$, the time at which h first comes within ε of its eventual limit. (Note that $t^*(\varepsilon)$ may be later than the present time T.) Suppose h^* is the estimate of h being used.

Let $\underline{t}^* = \underline{t}^*(\varepsilon,T) = \min_t \{h^*(t) - h^*(T) \leq \varepsilon\}$. Then $\underline{t}^*$ is our estimate for t^*, which may be interpreted as the beginning of the stable period if ε is suitably chosen.

When $T < t^*$, that is, stabilization has not yet occurred, then $\underline{t}^*$ should be fairly close to T. Thus when this phenomena is observed one should continue sampling and conclude that the breaking-in is still progressing. However when T surpasses t^* and continues growing then the estimates $\underline{t}^*$ should begin to recede from T and cluster about the fixed point t^*, and when this is perceived one should conclude that stabilization has begun.

Bernard McCabe

1033-BJM-jr B. J. McCabe

Attachment
Reference

REFERENCE

1. McCabe, B. J., "Decreasing Failure Rates and Some Related Statistical Tests," Bellcomm Technical Memorandum 68-1033-8, Case 101 (in preparation).

BELLCOMM, INC.

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From: B. J. McCabe

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